Graduate Level Courses

General Equilibrium in Economy on Riesz spaces

Pr. Belmesnaoui Aqzzouz
baqzzouz@hotmail.com

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Treillis de Banach: Théorie de Représentation de Kakutani

Pr. Karim Boulabiar
karim.boulabiar@gmail.com

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Analyse fonctionnelle et marchés financiers
Pr. Emmanuel Lepinette
lepinette@ceremade.dauphine.fr

Chapter 1. Modèle géométrique de marché financier avec friction en temps discret. Introduction et problèmes posés.

Chapter 2. Caractérisation de non arbitrage. Application de l'analyse fonctionnelle (Hahn-Banach) et convexe. Partie 1.

Chapter 3. Caractérisation de non arbitrage. Application de l'analyse fonctionnelle (Hahn-Banach) et convexe. Partie 2.

Chapter 4. Caractérisation duale des prix d'options Européennes. Application de la dualité convexe.

Chapter 5. Approche alternative en utilisant la valeur liquidative. Problématique basée sur un article récent de 2018-2019, en particulier un problème de fermeture d'un ensemble de L^0.

Workshop's Abstracts

Some Properties Of Weak Banach-Saks Operators.
Moulay Othman Aboutafail
EECOMAS Lab. ENSA, Université Ibn Tofail 1400 Kénitra, Morocco
aboutafail@yahoo.fr

Abstract: We establish necessary and sufficient conditions under which weak Banach-Saks operators are weakly compacts (resp. L-weakly compacts; resp. M-weakly compacts). As consequences, we give some interesting characterizations of order continuous norm (resp. reflexive Banach lattice).

Keywords: weak Banach-Saks, weakly compact operator, L-weakly compact operator, M-weakly com- pact operator, order continuous norm, positive Schur property and reflexive Banach space .

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Nonlinear Contractions Involving Simulation Functions In A Metric Space With A Partial Order

Hajer Argoubi
Carthge University, Tunisia
hajerargoubi1@gmail.com

Abstract: Very recently, Khojasteh, Shukla and Radenovic A new approach to the study of fixed point theorems via simulation functions, Filomat (2014) in press] introduced the notion of Z-contraction, that is, a nonlinear contraction involving a new class of mappings namely simulation functions. This kind of contractions generalizes the Banach contraction and unifies several known types of nonlinear contractions. In this paper, we consider a pair of nonlinear operators satisfying a nonlinear contraction involving a simulation function in a metric space endowed with a partial order. For this pair of operators, we establish coincidence and common fixed point results. As applications, several related results in fixed point theory in a metric space with a partial order are deduced.

Key words and phrases: Partial order, nonlinear contraction, coincidence point, common fixed point, simulation function.

Korovkin-type Approximation by operators in Riesz spaces via power series method.

Marwa Assili
University of Tunis El Manar, Tunisia
marwa.assili23@gmail.com

Abstract : In this Talk we present a Riesz spaces'version of the Korovkin-type Approximation by operators in the sense of power series method. That is, we try to extend Korovkin approximation theorems, obtained in 2016 and 2017 by Ozguc& Tas and Tas& Yurdakadim, respectively, for concrete classes of Banach Riesz spaces, to a wider class of Riesz spaces. Some applications are presented.

Several kinds of compactness of operators acting on Banach lattices and on lattice-normed spaces.

Youssef Azouzi
Carthge University, Tunisia
youssef.azouzi@ipest.rnu.tn

Abstract : This talk deals with compact operators. First we review some known results about compact operators, weakly compact operators and other classes. Then we present new results concerning un-compact operators and sequentially un-compact operators due to Kandič, Marabeh and Troitsky [4], and others concerning operators operators acting on lattice normed spaces and lattice normed vector lattices introduced recently by Aydın, Emelyanov, Erkurşun-Özcan and Marabeh. Finally we present some open questions and some answers that we obtained in a recent work jointly with Mohamed Amine Ben Amor.

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ru-Continuous Orthosymmetric Bilinear Maps On Vector Lattices

Elmiloud Chil
Tunis University, Tunisia
Elmiloud.chil@ipeit.rnu.tn

Abstract: The study of orthogonally additive polynomials is of interest both from the algebrain point of view and also from the point of view of infinite dimensional analysis, in particular the theory of holomorphic functions on infinite dimensional analysis. To the best of our knowledge the first mathematician interested in orthogonally additive polynomials was Sundaresan who obtained a representation theorem for polynomials on ℓ^p and on L^p. It is only recently that the class of such mappings has been getting more attention. We are thinking here about works on orthogonally additive polynomials and holomorphic functions and orthosymmetric multilinear mappings on different Banach lattices (see [5,7,10,13,18,21,22]), on $\mathbb{C}^*$-algebras (see [1,19]), on uniformly complete vector lattices (see[15,23,24]), on uniformly complete vector lattices with range space is separated convex bornological spaces (see [14]) and also on uniformly complete vector lattices taking values in Hausdorff topological vector spaces (see [3]). Note that except for approaches in [3,15,23,24] proofs of the aforementioned results are strongly based on the representation of these spaces as vector spaces of extended continuous functions. So they are not applicable to general vector lattices. Note also that the disadvantage of approaches in [3,15] the proofs are long quite involved not direct and uses the notion of s-power of a vector lattice which difficultly construct by using the tensor product in a vector lattice. That is why we need to develop new approaches. Actually, the innovation of this work consists in making a relationship between orthogonally additive polynomials and orthosymmetric multilinear mappings acts on uniformly complete vector lattices taking values in a Hausdorff topological vector spaces by using only the notion of the tensor product in a vector lattice. This leads to a constructive proofs of results in all papers mentioned above.

The notion of orthosymmetric mappings was introduced by Buskes and van Rooij [8]. A bilinear operator $b : E\times E \to F$ is called orthosymmetric if $|x|\wedge|y| = 0$ implies $b(x, y) = 0$ for arbitrary $x, y \in E$. Recall also that b is said to be symmetric if $b(x, y) = b(y, x)$. Now, we give a short historical note about orthosymmetric bilinear operator. In [8], Buskes and van Rooij proved that any positive othosymmetric bilinear operator is symmetric. Recently the first author in [11] proved that any order bounded orthosymmetric bilinear operator $b : E\times E \to F$ is symmetric. In the following paragraph, we intend to make some contribution of this area. We prove a generalization about this result to the more general setting of ru-continuous orthosymmetric bilinear maps. Before we pass to the details, we remark that some of the ideas of [3,4] are pursued; in the context of the current work, Theorem 14 in [4] allows the range space to be an Archimedean vectors lattice, whereas our range spaces are Hausdorff topological vector spaces (not necessarily vector lattices), proofs of results in [3] are heavily based on the s-power structure which we wish to avoid in this paper.

Keywords: Orthosymmetric multilinear map, homogeneous polynomial, vector lattice.

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Orthosymmetric Multilinear Maps And Polyorthomorphisms on Riesz Spaces

Abderraouf Dorai
University of Tunis El Manar, Tunisia
abderraouf.dorai@ipeiem.utm.tn

Abstract: We show that any multilinear orthosymmetric continuous map from an Archimedean Riesz space into a Hausdorff topological vector space is symmetric. Then, we introduce a new concept, namely that of polyorthomorphism of degree n, on a relatively uniformly complete f-algebra. We prove that, for a such Dedekind complete f-algebra E, the space Porth(nE) of all polyorthomorphisms of degree n is a Riesz space. We conclude with some interesting results about polyortho-morphisms of degree 2.

Orthosymmetric Spaces Over An Archimedean Vector Lattice.

Jamel Jaber
Carthge University, Tunisia
Jamel.jaber@free.fr

Abstract: We introduce and study the notion of orthosymmetric spaces over an Archimedean vector lattice as a generalization of Önite-dimentional Euclidean inner spaces. A special attention has been paid to linear operators acting on these spaces and their adjoints.

Lattice norms on the unitization of a truncated normed Riesz space

Hamza Hafsi
Tunis University, Tunisia
hafsi.hamza1@gmail.com

Truncated Riesz spaces was first introduced by Fremlin in the context of real-valued functions. An appropriate axiomatization of the concept was given by Ball. Keeping only the first Ball's Axiom (among three) as a definition of truncated Riesz spaces, the first named author and El Adeb proved that if E is truncated Riesz space then $E \oplus \mathbb{R}$ can be equipped with a non-standard structure of Riesz space such that E becomes a Riesz subspace of $E \oplus \mathbb{R}$ and the truncation of E is provided by meet with 1. In this talk, we assume that the truncated Riesz space E has a lattice norm $\|.\|$ and we give a necessary and sufficient condition for $E \oplus \mathbb{R}$ to have a lattice norm extending $\|.\|$. Moreover, we show that under this condition, the set of all lattice norms on $E \oplus \mathbb{R}$ extending $\|.\|$ has essentially a largest element $\|.\|_1$ and a smallest element $\|.\|_0$. Also, it turns out that any alternative lattice norm on $E \oplus \mathbb{R}$ is either equivalent to $\|.\|_1$ or equals $\|.\|_0$. As consequences, we show that $E \oplus \mathbb{R}$ is a Banach lattice if and only if E is a Banach lattice and we get a representation's theorem sustained by the celebrate Kakutani's Representation Theorem.

Almost L-limited sets

Jawad Himichane
Mouley Ismail University, Meknes, Morocco
hm1982jad@gmail.com

Abstract: We introduce and study new class of sets (almost L-limited sets). Also, we introduce new concept of property in Banach lattice (almost Gelfan-Phillips property) and we characterize this property using almost L-limited sets. On the other hand, we introduce the class of disjoint limited completely continuous operators which is a largest class than that of limited completely continuous operators, we characterize this class of operators and we study some of its properties.

The Wickstead Problem

Naoual KOUKI
Tunisia
naoual.kouki@gmail.com

Abstract: Wickstead raised the problem of automatic order boundedness of all band pre- serving linear operators. The answer depends on the vector lattice in which the operators in question acts. There are many results that deals with this subject. In this work we focus our attention on the Wickstead Problem for the case of Archimedean semiprime f-algebras by focusing on the commutativity of the ordered algebra of all band preserving operators. More precisely, we prove that if A an Archimedean semiprime f-algebra, then the collection B(A) of all band preserving operators on A are automatically order bounded if and only any derivation on B(A) is null.

Unitization of a lattice ordered ring with a truncation

Mounir Mahfoudhi
University Of Tunis El Manar, Tunisia
mahfoudhi.mounir@hotmail.fr

Abstract: Let R be a lattice ordered ring along with a truncation in the sense of Ball. We give a necessary and sufficient condition on R for its unitization $R \oplus \mathbb{Q}$ in order to be again a lattice ordered ring. Also, we shall see that $R \oplus \mathbb{Q}$ is a lattice ordered ring for at most one truncation. A special attention is paid to the Archimedean case. More precisely, we shall identify the unique truncation on an Archimedean ℓ-ring R which makes $R \oplus \mathbb{Q}$ into a lattice ordered ring.

AM-compact operators on normed Riesz spaces

Radhouene Moulahi
Sfax University, Tunisia
piborri@gmail.com

Abstract: We introduce and study the graph lattice norm. Also we give some new results concerning the domination problem by AM-compact (relatively compact) operators between normed Riesz spaces.

Martingale convergence via the square function in vector lattices

Kawter Ramadane
Morocco
ramdanekawtar@gmail.com

Abstract: The main purpose of the talk is to give a vector lattice version of a Theorem by Burkholder about convergence of martingales. The proof is based on a vector lattice analogue of Austin's sample function theorem, proved recently by Grobler, Labuschagne and Marraffa and on a complete vector lattice which do not belong to the space.